MATH 598: Random matrix theory

Winter 2022

Professor: Elliot Paquette, elliot.paquette@mcgill.ca.

Course schedule: TBA.

Textbooks: There is no course textbook. Material will be drawn from multiple sources, in particular the books listed below.

1. *High–dimensional probability: an introduction with applications in data science.* **Roman Vershynin**. Available online via the McGill Library. Draft version available online at the author's website.
2. *Random matrix theory for machine learning.* **Romain Couillet and Zhenyu Liao**. Draft version available online at the author's website.
3. *Topics in random matrix theory.* **Terence Tao**. Draft version available online at the author's website.
4. *Operator limits of random matrices.* **Diane Holcomb** Lecture notes available from the author's website.

Prerequisite: Probability (Math 356 or equivalent: in particular central limit theorems, convergence in law, and convergence in probability). Complex analysis (Math 249 or equivalent: in particular cauchy's integral theorem).

Corequisite: Stochastic calculus (familiarity with brownian motion, Itô's formula, and stochastic differential equations will help with the final quarter of the class – we will have a crash course in the material, but familiarty from outside is strongly suggested. See the first 5 chapters of Oksendal's "Stochastic differential equations" Oksendal @ McGill library for more).

Topics:

- Matrix analyis
  - Norms
  - Eigenvalue inequalities and Cauchy interlacing
  - The resolvent and eigenvalue perturbation
- Concentration of measure
  - Subgaussian/subexponential random variables, Lipschitz concentration
  - Bernstein inequality & the Hanson-Wright inequality

  - Concentration of the norm of a random matrix
- Classical ensembles
  - 4 canonical random matrix ensembles
  - The joint distribution of eigenvalues, and the Weyl integration formula
  - The orthogonal group and concentration of measure.
- The resolvent method and deterministic equivalents
  - The semicircle law & Marchenko–Pastur law
  - The fixed point equation for sample covariance matrices
  - Outliers and spiked ensembles.
- Crash course in Brownian motion & Stochastic calculus
- Tridiagonalization, and the local laws
  - The sine point process
  - The Airy point process
  - The Baik-Ben Arous-Péché transition

Grading Scheme

Your grade will be computed from two components. Four homeworks, corresponding to the first four bullet points, and a final project. The final project may be replaced with a take-home exam, which you will have a week to complete.

- Marked homework assignments: 50%
- Final project: 50%

Solutions will be judged on mathematical correctness and completeness. Students are encouraged to work in groups, and may do so for their final assignments and projects (max 3). One form of the final project is a 'read and summarize,' in which you read a paper in random matrix theory from a suggested list and summarize its contents.

Language

In accord with McGill University's Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

Conformément à la Charte des droits de l'étudiant de l'Université McGill, chaque étudiant a le droit de soumettre en français ou en anglais tout travail écrit devant être noté (sauf dans le cas des cours dont l'un des objets est la maîtrise d'une langue).

Academic Integrity

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