# MATH 598: High-Dimensional Probability

Fall 2023

Professor: Elliot Paquette, elliot.paquette@mcgill.ca.

Course schedule: Monday and Wednesday, 2:35 pm - 3:55 pm.

Textbooks: There is no official textbook for the course. The following are good references that could be used as a supplement:

*High-dimensional probability: An introduction with Applications in Data Science*. **Vershynin**. Available online via the Author's website. This is an approachable book which covers many of the core topics. It is occasionally fails to go into sufficient detail. The course will be aligned in great part with the first 8 chapters.

*High-dimensional statistics: a non-asymptotic viewpoint.* **Wainwright**. Tome containing much of the same math as Vershynin but with much deeper statistical connections. Very likely to prove surprising every time you open it.

This course develops the theory of high-dimensional probability: random vectors and matrices and the mathematics of how these transform when one applies transformations (especially convex transformations) to them, such as norms and seminorms, eigenvalue maps, and others. This reveals fundamental geometric properties of normed spaces and convex sets in high dimensions, and it is also deeply connected to modern application in statistics, computer science and data science. Topics covered will be: concentration of measure, net arguments and norm bounds, Gaussian processes and Gaussian concentration, chaining, and many applications of the above.

This is a preliminary outline of the course content.

- Concentration of measure
  - The geometric basis of concentration of measure
    - Thin polytopes, "hyperbolicity," Pajor theorem
    - Approximate Caratheodory Theorem
  - Tails of random variables
  - Subgaussian random variables
  - Subexponential random variables

- Norms and nets
  - The norm of a random matrix.
  - Covariance matrix estimation
  - The Marchenko-Pastur law, and the low-dimensional data
  - The smallest singular value problem
- Gaussian processes
  - Gaussian processes
  - Sudakov and Gordon inequalities
  - Gaussian width
  - Lipschitz concentration
  - Log-sobolev inequalities and beyond
- Chaining
  - Maxima of processes
  - Dudley's Entropy integral
  - Talagrand's generic chaining
- Topics
  - Dvoretzky-Milman
  - Applications in statistics

Prerequisite: Math 356 Probability or equivalent. Familiarity with norms, vector spaces, convexity.

Grading Scheme

**Caveat.** If the resources provided by the university (in the form of teaching assistants and graders) is inadequate, the grading scheme may be modified.

Your grade will be calculated as the maximum of the following two formulas:

Formula:

- Marked homework assignments: 50%
- Final: 50%

Solutions will be judged on mathematical correctness, completeness, and also on clarity of exposition.

The book *Mathematical writing for undergraduate students* is a useful resource for learning the basics of clear mathematical writing. It is available at via the McGill library catalogue at McGill Library Link.

Students may work in groups, but must each write up their assignment solutions on their own.

McGill's "student rights and responsibilities" web page has this to say:

> Peer learning should be encouraged, since it helps students learn to teach. Instructors should explain effective peer teaching strategies such as working in pairs, sharing comments on work, and brainstorming solutions to problems in groups. Sharing com-

> pleted work is not an acceptable peer learning technique. If a student has copied the answers of another student, the incident must be documented and the material sent to the course instructor, who will contact the appropriate disciplinary officer.
> –From Official McGill Policy

## Language

In accord with McGill University's Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

Conformément à la Charte des droits de l'étudiant de l'Université McGill, chaque étudiant a le droit de soumettre en français ou en anglais tout travail écrit devant être noté (sauf dans le cas des cours dont l'un des objets est la maîtrise d'une langue).

## Academic Integrity

McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see Official McGill page for more information). This, in particular, excludes usage of websites such as Chegg, Course Hero, Bartleby, or Scribd for the purpose of answering assigment and exam questions, which will be strictly enforced.

L'université McGill attache une haute importance à l'honnêteté académique. Il incombe par conséquent à tous les étudiants de comprendre ce que l'on entend par tricherie, plagiat et autres infractions académiques, ainsi que les conséquences que peuvent avoir de telles actions, selon le Code de conduite de l'étudiant et des procédures disciplinaires (pour de plus amples renseignements, veuillez consulter le site.